MULTIPLE CHOICE. Consider the DEs **SCORE: _____ / 3 PTS**

[1] $x''y - y^2 = 2x$ [2] $(5r+1)\dfrac{dr}{d\theta} = 2\theta + 1$ [3] $(5w+1)du + (\ln w - 3u)dw = 0$

(where $w$ is the independent variable)

Which of the DE above are linear ? Circle the correct answer below.

(a) none are linear (b) only [1] is linear (c) only [2] is linear (d) only [3] is linear

(e) only [1] & [2] are linear (f) only [1] & [3] are linear (g) only [2] & [3] are linear (h) all are linear

3

Consider the autonomous DE $y' = (y+1)^2(y-3)$.

**SCORE: ____ / 7 PTS**

[a] Find all equilibrium solutions of the DE and classify each as stable, unstable or semi-stable.

$(y+1)^2(y-3)=0 \longrightarrow y=-1,3$

(1) $y'>0$

$3$

$y=3$ UNSTABLE (1)

(1) $y'<0$

$-1$

(1) $y'<0$

$y=-1$ SEMI-STABLE (1)

[b] If $y=f(x)$ is a solution of the DE such that $f(-4)=1$, what is $\lim_{x\to\infty} f(x)$ ? HINT: Sketch a possible graph of $y=f(x)$.

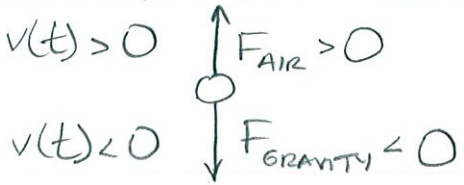
$-1$ (1)

[c] If $y=g(x)$ is a solution of the DE such that $g(5)=-2$, what is $\lim_{x\to\infty} g(x)$ ?

$-\infty$ (1)

Write a differential equation for the velocity $v(t)$ of a falling object if the air resistance is proportional to the square of the velocity. Assume that $v(t) > 0$ corresponds to the object moving upward, $v(t) < 0$ corresponds to the object moving downward. (NOTE: This is NOT the same problem as in the homework.)

**SCORE: ____ / 3 PTS**




$$m\frac{dv}{dt} = -mg + kv^2$$

(1) (1) (1)

MUST HAVE CORRECT SIGN

FILL IN THE BLANKS.

**SCORE: ____ / 3 PTS**

[a] The order of the DE $y^{10} - y^7 y^{(4)} = (x^6 + y''')^5$ is 4. (1)

[b] If $y = \sqrt{x+4}$ is a solution of the DE $y'' = f(x, y, y')$, the largest possible interval of definition is $(-4, \infty)$. (2)

$y' = \frac{1}{2}(x+4)^{-\frac{1}{2}}$ $y'' = -\frac{1}{4}(x+4)^{-\frac{3}{2}}$

NOTE: -4 IS NOT IN THE INTERVAL

Consider the IVP $y' = 10x - 5y, \quad y(1) = -4$.

**SCORE: _____ / 5 PTS**

Use Euler's method with $h = 0.2$ to estimate $y(1.4)$.

$$y(1.2) \approx y(1) + y'(1)(0.2) = -4 + (10(1) - 5(-4))(0.2)$$

$$= -4 + 30(0.2) = 2 \quad (2)\ (1)$$

$$y(1.4) \approx y(1.2) + y'(1.2)(0.2) \approx 2 + 2(0.2) = 2.4$$

$(1)\ (1)$

What does the Existence & Uniqueness Theorem tell you about the IVP $(\cos x)y' - y^{\frac{3}{2}} = 0, \quad y(\frac{\pi}{4}) = 0$ ? **SCORE:** _____ / 3 PTS

Justify your answer properly, but briefly.

(1/2) $y' = \frac{y^{\frac{3}{2}}}{\cos x} = f$, so (1) $f_y = \frac{\frac{3}{2}y^{\frac{1}{2}}}{\cos x}$ WHICH IS NOT DEFINED/CONTINUOU AROUND $(\frac{\pi}{4}, 0)$ WHERE $y < 0$ (1/2)

SO E+U TELLS US NOTHING. (1)

Consider the DE $x^2\dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} + 2y = x^3$.

**SCORE: ____ / 6 PTS**

[a] Is $y = x^3 + Ax^2 + Bx$ a family of solutions of the DE ?

$y' = 3x^2 + 2Ax + B$

$y'' = 6x + 2A$ (1)

$x^2y'' - 2xy' + 2y = x^2(6x+2A) - 2x(3x^2+2Ax+B) + 2(x^3+Ax^2+Bx)$ (1) (1)

$= 2x^3$ ($\frac{1}{2}$)

NO ($\frac{1}{2}$)

[b] If the answer to [a] is "YES", solve the IVP consisting of the DE and the initial conditions $y(1) = 3, \quad y'(1) = -1$.
If the answer to [a] is "NO", skip this part.